EBS 301 CALCULUS
SAMPLE QUESTIONS

1a. Show from first principles that the derivative of a constant is zero.

Marking scheme

Let $f(x) = c$ where c is a constant

$f(x) = cx^0$ (since $x^0 = 1$)

$\Longrightarrow f(x+h) = c(x+h)^0 = c$

$f'(x) = \lim_{h\to 0}[\frac{f(x+h)-f(x)}{h}]$

$f'(x) = \lim_{h\to 0}\frac{c-c}{h} = \lim_{h\to 0}\frac{0}{h} = \lim_{h\to 0} 0 = 0$

Hence the derivative of any constant is zero

1b. A ball is thrown vertically upwards. Its height h(m) at time t s is given by

$h = 5 + 30t - 5t^2$. Find

i) The velocity of the ball at time $t = 2\ s$

ii) The maximum height reached.

Marking scheme

i) $h = 5 + 30t - 5t^2$

Velocity, $v = \frac{dh}{dt} = 30 - 10t$
At $t = 2\ s$
$v = 30 - 10(2)$
$= 10\ ms^{-1}$

ii). At the maximum height, velocity $v = 0$

$\Rightarrow 30 - 10t = 0$
$\Rightarrow t = 3\ s$
For the maximum height, $h = 5 + 30(3) - 5(3)^2$
$= 50$
∴The maximum height reached is 50 m.

1c. Find the equation of the normal to the curve $x^2 + xy + 2y^2 = 8$ at the point $(-3,1)$.

Marking scheme

$x^2 + xy + 2y^2 = 8$

$2x + x\frac{dy}{dx} + y(1) + 4y\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 4y\frac{dy}{dx} = -2x - y$

$(x + 4y)\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx} = \frac{-2x - y}{x + 4y}$

At the point $(-3,1)$

$\frac{dy}{dx} = \frac{-2(-3) - 1}{-3 + 4(1)} = \frac{5}{1} = 5$

$\therefore$ Gradient of a tangent to the curve is 5.

Gradient of the normal to the curve is $-\frac{1}{5}$

Using the point $(-3,1)$, the equation of the normal is:

$y - 1 = -\frac{1}{5}(x + 3)$

$5y - 5 = -x - 3$

$5y + x - 2 = 0$

OBJECTIVES

1. Evaluate $\int_0^1 (x^3+1)^2 dx$

A. $\frac{9}{14}$
B. $\frac{11}{14}$
C. $\frac{23}{14}$
D. $\frac{31}{14}$

2. If $\int_2^a (2x+2)dx = 8\,, a > 0$, determine the value of a.

A. 2
B. 3
C. 6
D. 8

3. If $y = \int (3x^2+4x)dx$ and $y = 7$ when $x = 1$, find y.

A. $y = x^3 + 2x^2 + 7$
B. $y = x^3 + 2x^2 - 7$
C. $y = x^3 + 2x^2 - 4$
D. $y = x^3 + 2x^2 + 4$

4.Determine the turning point of the curve $y = x^2 - 4x + 1$.
A. (2,-3)
B. (2,-5)
C. (2,3)
D. (2,5)

1. If $y = 3x^2 - 5, x = 2$ and $\Delta x = 0.1$, find Δy, where Δx and Δy are the increments in x and y.
A. 1.23
B. 1.3
C. 1.4
D. 1.6